1η Μεθ: k : $P(\text{απορ } H_0 / \theta=\theta_0)=a$

2η Μεθ: k : $k_1 P(\text{απορ } H_0 / H_0 \text{ αληθ}) = k_0 \cdot P(\text{αποδ } H_0 / H_a \text{ αληθ})$

3η Μεθ: $k = \dfrac{\ell(\theta, d_0)\, \pi(\theta_a)}{\ell(\theta, d_a)\cdot \pi(\theta_0)} = \dfrac{k_0}{k_1}\cdot\dfrac{\pi(\theta_a)}{\pi(\theta_0)}$

Παράδειγμα 1

$X_1, X_2, \ldots, X_n \sim N(\theta, 100)$

$n=100$

$H_0: \theta=75$ v $H_a: \theta=78$

i) Ισχυρότερο τεστ 5%.

ii) MinMax $\ell(\theta_0, d_a)=3$ και

MinMax $\ell(\theta_a, d_0)=1$

iii) Bayes τεστ $\ell(\theta_0, d_a)=\ell(\theta_a, d_0)$

$\pi(\theta_0)=\pi(\theta_a)$

Να βρεθεί το a και το γ.

Απάντηση

i) $\dfrac{\prod f(x_i|\theta_0)}{\prod f(x_i|\theta_a)} \le k \Rightarrow \dfrac{e^{-\frac{1}{2\sigma^2}\sum(x_i-\theta_0)^2}}{e^{-\frac{1}{2\sigma^2}\sum(x_i-\theta_a)^2}} \le k \Rightarrow$

$\Rightarrow e^{-\frac{1}{2\sigma^2}\left\{-2\theta_0\sum x_i + n\theta_0^2 - 2\theta_a \sum x_i - n\theta_a^2\right\}} \le k$ (1)

$\Rightarrow -\dfrac{1}{2\sigma^2}\left(2\sum x_i(\theta_a-\theta_0) + n(\theta_0^2-\theta_a^2)\right) \le \log k \Longrightarrow$

$2\Sigma x_i(\theta_1-\theta_0)+n(\theta_0^2-\theta_1^2)\geq -2\sigma^2\log k \Rightarrow$

$\Rightarrow \Sigma x_i(\theta_1-\theta_0)\geq -\sigma^2\log k-\frac{n}{2}(\theta_0^2-\theta_1^2)\Rightarrow$

$$\Rightarrow \Sigma x_i \geq \frac{-\sigma^2\log k - n/2(\theta_0^2-\theta_1^2)}{\theta_1-\theta_0}=k_1$$

k : τ/ω η πιθανοτητα σφ. τυπου I

$P(\text{απορ } H_0 / H_0 \text{ αληθ}) = a \Rightarrow P(\Sigma x_i \geq k_1 / X_i \sim N(\theta_0,\sigma^2)\ \forall i) = a$

$\Rightarrow P(\Sigma x_i \geq k_1 / \Sigma x_i \sim N(n\theta_0, n\sigma^2)) = a \Rightarrow$

$$\Rightarrow P\left(\frac{\Sigma x_i - n\theta_0}{\sqrt{n\sigma^2}} \geq \frac{k_1-n\theta_0}{\sqrt{n\sigma^2}} / H_0 \text{ αληθ}\right) = a$$

$$\text{οπου } Z_a = \frac{k_1-n\theta_0}{\sqrt{n\sigma^2}} \Rightarrow k_1=\sqrt{n\sigma^2}Z_a+n\theta_0$$

για την ισχύ του τεστ:

$\gamma = P(\text{απορ } H_0 / H_a \text{ αληθ}) = P(\Sigma x_i \geq k_1 / \Sigma x_i \sim N(n\theta_a, n\sigma^2)) =$

$$= P\left(\frac{\Sigma x_i - n\theta_a}{\sqrt{n\sigma^2}} \geq \frac{k_1-n\theta_a}{\sqrt{n\sigma^2}} / \Sigma x_i \sim N(n\theta_a, n\sigma^2)\right) =$$

$$= 1-\Phi\left(\frac{k_1-n\theta_1}{\sqrt{n\sigma^2}}\right).$$

ii) $k_1 \cdot P(\Sigma x_i \geq k_1 / x_i \sim N(\theta_0,\sigma^2)) = k_0 \cdot P(\Sigma x_i < k_1 / x_i \sim N(\theta_a,\sigma^2))$

$$3\cdot P\left(\frac{\Sigma x_i - n\theta_0}{\sqrt{\sigma^2 n}} \geq \frac{k_1-n\theta_0}{\sqrt{\sigma^2 n}} / H_0 \text{ αληθ}\right) =$$

$$= 1\cdot P\left(\frac{\Sigma x_i - n\theta_a}{\sqrt{\sigma^2 n}} < \frac{k_1-n\theta_a}{\sqrt{\sigma^2 n}} / H_a \text{ αληθής}\right) \Rightarrow$$

$$\Rightarrow 3\cdot\left(1-\Phi\left(\frac{k_1-n\theta_0}{\sqrt{\sigma^2 n}}\right)\right) = 1\cdot\Phi\left(\frac{k_1-n\theta_a}{\sqrt{\sigma^2 n}}\right) \quad (*)$$

Αρκεί, να βρώ το k_1 για το οποίο επαληθευεται η εξίσωση (*). Η συνθήκη (*) θα δίνεται στις εξετάσεις ου συνεπάγεται ενα συγκεκριμενο k_1. Στην προκειμένη περίπτωση το $k_1 = 7680$

Απαντηση στο ερώτημα: Ποιο το επιπεδο σημαντικοτητας;

$\alpha = P(\Sigma X_i \geq k_1 \mid X_i \sim N(\theta_0, \sigma^2))$
και δείξαμε πριν, ότι τελικά:
$\alpha = 1 - \Phi\left(\frac{k_1 - n\theta_0}{\sqrt{n\sigma^2}}\right)$ για k_1 δοθέν.

Για την ισχύ

$$\gamma = 1 - \beta = 1 - \Phi\left(\frac{k_1 - n\theta_1}{\sqrt{n\sigma^2}}\right) \text{ για } k_1 \text{ δοθέν}$$

iii) Εάν $\lambda = e^{-\frac{1}{2\sigma^2}\{\Sigma(x_i-\theta_0)^2 - \Sigma(x_i-\theta_a)^2\}} \leq k = \frac{k_0}{k_1} \cdot \frac{\pi(\theta_a)}{\pi(\theta_0)}$

$$\Rightarrow \lambda \leq \frac{k_0}{k_1} \cdot \frac{\pi(\theta_a)}{\pi(\theta_0)} = 1 \Rightarrow -\frac{1}{2\sigma^2}\{\Sigma(x_i-\theta_0)^2 - \Sigma(x_i-\theta_a)^2\} \leq 0$$

$$\Rightarrow \Sigma(x_i-\theta_0)^2 - \Sigma(x_i-\theta_a)^2 \geq 0 \Rightarrow$$
$$\Rightarrow -2\Sigma x_i\theta_0 + n\theta_0^2 + 2\Sigma x_i\theta_a - n\theta_a^2 \geq 0 \Rightarrow$$
$$\Rightarrow 2\Sigma x_i(\theta_a - \theta_0) \geq n\theta_a^2 - n\theta_0^2 \Rightarrow$$
$$\Rightarrow \Sigma x_i \geq \frac{n(\theta_a^2 - \theta_0^2)}{2(\theta_a - \theta_0)} = \frac{n(\theta_a + \theta_0)}{2} = \frac{\cancel{100}^{50}(78+75)}{\cancel{2}} = \underset{k_2}{7650}$$

Όπως, και πριν

$$\alpha = P(\text{απορ } H_0 \mid H_0 \text{ αληθής}) = \ldots = 1 - \Phi\left(\frac{k_2 - n\theta_0}{\sqrt{n\sigma^2}}\right)$$
και
$$\gamma = P(\text{απορ } H_0 \mid H_a \text{ αληθής}) = \ldots = 1 - P(\Sigma x_i \leq k_2 \mid \Sigma x_i \sim N(n\theta_a, n\sigma^2))$$

Παράδειγμα 2

$X_1, X_2, \ldots, X_n$ τδ από την $Εκθ(\theta)$.
Να ελέγξετε τις υποθέσεις
$H_0: \theta = \theta_0 = 1$ v $H_1: \theta = \theta_1 = 2$
i) ισχυρότατο τεστ $a = 5\%$
ii) Minmax: $L(\theta_0, d_1) = L(\theta_1, d_0)$
iii) $\frac{\pi(\theta_1)}{\pi(\theta_2)} = \frac{1}{2}$

ΛΥΣΗ

$$\lambda = \frac{\prod_{i=1}^{n} \theta_0 e^{-\theta_0 x_i}}{\prod_{i=1}^{n} \theta_1 e^{-\theta_1 x_i}} = \frac{\theta_0^n}{\theta_1^n} \cdot \frac{e^{-\theta_0 \sum_{i=1}^{n} x_i}}{e^{-\theta_1 \sum_{i=1}^{n} x_i}} = \frac{\theta_0^n}{\theta_1^n} \cdot e^{-\Sigma x_i (\theta_0 - \theta_1)}$$

i) $\lambda \le k \Rightarrow \frac{\theta_0^n}{\theta_1^n} \cdot e^{-\Sigma x_i}(\theta_0 - \theta_1) \le k \Rightarrow$

$$\Rightarrow e^{-\Sigma x_i (\theta_0 - \theta_1)} \le \frac{k\theta_1^n}{\theta_0^n} \Rightarrow$$

$$\Rightarrow -\Sigma x_i (\theta_0 - \theta_1) \le \log\left(\frac{k\theta_1^n}{\theta_0^n}\right) \Rightarrow$$

$$\Rightarrow \Sigma x_i \le \frac{\log\left(\frac{k\theta_1^n}{\theta_0^n}\right)}{\theta_1 - \theta_0} = k_1 \quad (*)$$

ή ισοδύναμα $\boxed{\Sigma x_i \le k_1}$

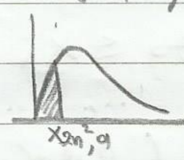
i) $\alpha = P(\Sigma x_i \le k_1 \,/\, X_i \sim \text{Εκθ}(\theta_0), \forall i) =$

$= P(\Sigma x_i \le k_1 \,/\, X_i \sim \text{Γάμμα}(1, \frac{1}{\theta_0})) =$

$= P(\Sigma x_i \le k_1 \,/\, X_i \sim \text{Γάμμα}(n, \frac{1}{\theta_0})) =$

$= P(2\theta_0 \Sigma x_i \le 2\theta_0 k_1 \,/\, 2\theta_0 X_i \sim \chi^2_{2n}) \Rightarrow$

$\Rightarrow 2\theta_0 k_1 = \chi^2_{2n,\alpha}$



Για την ισχύ του τεστ:

$\gamma = P(\Sigma x_i \le \frac{\chi^2_{2n,a}}{2\theta_0} \,/\, X_i \sim \text{Εκθ}(\theta_1)) =$

$= P(\Sigma x_i \le \frac{\chi^2_{2n,a}}{2\theta_0} \,/\, \Sigma X_i \sim \text{Γάμμα}(n, \frac{1}{\theta_1})) =$

$= P(2\theta_1 \Sigma x_i \le \frac{\chi^2_{2n,a}}{2\theta_0} \cdot 2\theta_1 \,/\, 2\theta_1 \Sigma X_i \sim \chi^2_{2n}) =$

$= F_{\chi^2_{2n}}\left(\frac{\theta_1}{\theta_0} \cdot \chi^2_{2n,\alpha}\right)$. (το οποίο θα δίνεται στις εξετάσεις).

ii) Το k_1 προσδιορίζεται έτσι ώστε

$\cancel{d(\theta_0, d_1)} \cdot P(\text{απορ. } H_0 \mid H_0 \text{ αληθ.}) = \cancel{d(\theta_1, d_0)} \cdot P(\text{αποδ } H_0 \mid H_0 \text{ αληθ})$

$P(\Sigma x_i \leq k_1 \mid x_i \sim \Gamma\alpha\mu\mu\alpha(1, \frac{1}{\theta_0})) = P(\Sigma x_i > k_1 \mid x_i \sim \Gamma\alpha\mu\mu\alpha(1, \frac{1}{\theta_1}))$

$P(\Sigma x_i \leq k_1 \mid \Sigma x_i \sim \Gamma\alpha\mu\mu\alpha(n, \frac{1}{\theta_0})) = P(\Sigma x_i > k_1 \mid \Sigma x_i \sim \Gamma\alpha\mu\mu\alpha(n, \frac{1}{\theta_1}))$

$P(2\theta_0 \Sigma x_i \leq 2\theta_0 k_1 \mid 2\theta_0 \Sigma x_i \sim \chi^2_{2n}) = P(2\theta_1 \Sigma x_i > 2\theta_1 k_1 \mid 2\theta_1 \Sigma x_i \sim \chi^2_{2n})$

$F_{\chi^2_{2n}}(2\theta_0 k_1) = 1 - F_{\chi^2_{2n}}(2\theta_1 k_1)$ (αν μας δίνεται σε ποιο κ. λύνεται η συνθήκη αυτή, αν όχι τότε το αφήνουμε σε αυτή τη μορφή)

iii) $k = \frac{1}{2}$, (αφού $d(\theta_0, d_0) = d(\theta_1, d_0)$ και $\frac{\Pi(\theta_1)}{\Pi(\theta_0)} = \frac{1}{2}$)

Άρα, από την (*)

$$\Sigma x_i \leq \frac{\log\left(\frac{1}{2}\frac{2^n}{1^n}\right)}{2-1} \Rightarrow \Sigma x_i \leq \log 2^{n-1} \Rightarrow$$

$\Rightarrow \Sigma x_i \leq (n-1)\log 2$, η κρίσιμη περιοχή

$$a = P(\Sigma x_i \leq (n-1)\log 2 \mid x_i \sim \Gamma\alpha\mu\mu\alpha(1, \tfrac{1}{\theta_0})) =$$

$$= P(2\theta_0 \leq x_i \leq 2\theta_0(n-1)\log 2) =$$

$$= F_{\chi^2_{2n}}(2\theta_0(n-1)\log 2)$$

Ισχύς

$$\gamma = P(\Sigma x_i \leq (n-1)\log 2 \mid x_i \sim \Gamma\alpha\mu\mu\alpha(1, \tfrac{1}{\theta_1})) =$$

$$= P(2\theta_1 \Sigma x_i \leq 2\theta_1(n-1)\log 2)$$

Άσκηση

$X_1, X_2, \ldots, X_n$ τδ με $n=10$ από Poisson(θ)

Να ελέγξουμε $H_0: \theta=\theta_0=0.5$ vs $H_1: \theta=\theta_1=1$, και να βρεθεί:

i) Ισχυρότατο τεστ με $3\% < \alpha < 5\%$

ii) Minmax τεστ: $L(\theta_0, d_1) = 1{,}65$ και $L(\theta_1, d_0) = 1$

iii) Bayes τεστ: $L(\theta_1, d) = 2$ και $L(\theta_0, d_1) = 1$

$\Pi(\theta_0) = \frac{1}{3}$ και $\Pi(\theta_1) = \frac{2}{3}$

Λύση

$$\lambda := \frac{\prod_{i=1}^{n} \frac{\theta_0^{x_i} \cdot e^{-\theta_0}}{x_i!}}{\prod_{i=1}^{n} \frac{\theta_1^{x_i} \cdot e^{-\theta_1}}{x_i!}} = \left(\frac{\theta_0}{\theta_1}\right)^{\Sigma x_i} \cdot e^{-n(\theta_0-\theta_1)} \leq k$$

$$\left(\frac{\theta_0}{\theta_1}\right)^{\Sigma x_i} \leq k \cdot e^{n(\theta_0-\theta_1)} \Rightarrow \Sigma x_i \log\left(\frac{\theta_0}{\theta_1}\right) \leq \log k + n(\theta_0-\theta_1)$$

$$\Rightarrow \Sigma x_i \geq \frac{\log\left(k \cdot e^{n(\theta_0-\theta_1)}\right)}{\log \frac{\theta_0}{\theta_1}} = k^* \quad (*)$$

i) Ισχυρότατο τεστ:

$$\alpha = P(\Sigma x_i \geq k^* \mid x_i \sim \text{Poisson}(\theta_0 = 0.5)) =$$

$$= P(\Sigma x_i \geq k^* \mid \Sigma x_i \sim \text{Poisson}(n \cdot \theta_0 = 5)) =$$

$$= 1 - P(\Sigma x_i < k^* \mid \Sigma x_i \sim \text{Poisson}(5)) =$$

$$= 1 - P(\Sigma x_i \leq k^* - 1 \mid \Sigma x_i \sim \text{Poisson}(5))$$

Ψάχνουμε $\alpha \in (3\%, 5\%)$ από τον πίνακα της Poisson κατανομής. Ένα τέτοιο α στη στήλη 5.0 είναι το 0,9682. Επομένως, $k^*-1 = 9 \Rightarrow k^* = 10$

Άρα, $\Sigma x_i \geq 10$ η κρίσιμη περιοχή

Η ισχύς:

$\gamma = P(\Sigma x_i \geq 10 \,/\, \Sigma x_i \sim Poisson(n\theta_1 = 10)) =$

$= 1 - P(\Sigma x_i < 10 \,/\, \Sigma x_i \sim Poisson(10)) =$

$= 1 - P(\Sigma x_i \leq 9 \,/\, \Sigma x_i \sim Poisson(10)) =$

$= 1 - 0{,}4579.$

iii) $k = \frac{2}{1} \cdot \frac{2/3}{1/3} = 4$

Άρα η σχέση (*) δίνει

$$\Sigma x_i \geq \frac{\log\left(4 \cdot e^{10\cdot(0,5-1)}\right)}{\log \frac{0.5}{1}} = \frac{\log(4 \cdot e^{-5})}{-\log 2} =$$

$$= \frac{2\log 2 - 5}{-\log 2} = -2 + \frac{5}{\log 2} = 5.2$$

Άρα, $\Sigma x_i \geq 5.2 \iff \Sigma x_i \geq 6.$

$\alpha = P(\Sigma x_i \geq 6 \,/\, \Sigma x_i \sim Poisson(5)) =$

$= 1 - P(\Sigma x_i < 6 \,|\, \Sigma x_i \sim Poisson(5)) =$

$= 1 - P(\Sigma x_i \leq 5 \,|\, \Sigma x_i \sim Poisson(5)) =$

$= 1 - 0{,}6160$

Η ισχύς:

$\gamma = P(\Sigma x_i \geq 6 \,|\, \Sigma x_i \sim Poisson(10)) =$

$= 1 - P(\Sigma x_i \leq 5 \,|\, \Sigma x_i \sim Poisson(10))$

$= 1 - 0{,}0671$

ii) $1{,}65 \cdot P(\Sigma X_i \geq k^* / X_i \sim Poisson(0{,}5)) = 1 \cdot P(\Sigma X_i < k^* / X_i \sim P(1))$

$1.65\, P(\Sigma X_i \geq k^* / \Sigma X_i \sim Poisson(5)) = 1 \cdot P(\Sigma X_i < k^* / \Sigma X_i \sim P(10))$

$1.65 (1 - P(\Sigma X_i \leq k^* - 1 / \Sigma X_i \sim P(5)) = P(\Sigma X_i \leq k^* - 1 / \Sigma X_i \sim P(10))$

Η λύση θα βρεθεί για εκείνο το k^* που την επαληθεύει

Μέσω δοκιμών

- $k^* - 1 = 0 \Rightarrow k^* = 1$
- $k^* - 1 = 1 \Rightarrow k^* = 2$
- $k^* - 1 = 2 \Rightarrow k^* = 3$

⋮

- $k^* - 1 = 8 \Rightarrow k^* = 9$

⋮

Για $k^* - 1 = 0 \Rightarrow k^* = 1$ είναι:

$1{,}65 \cdot (1 - 0.0067) \neq 0$

$1{,}65 \cdot (1 - 0{,}0404) = 0.005$

$1{,}65 \cdot (1 - 0{,}1247) = 0.0028$

⋮

Μετά από επαληθευσεις θα βρουμε ότι $k^* = 8$.
Αυτή που θα ικανοποιεί την εξίσωση παραπάνω θα είναι και μοναδική

Tables of the Poisson Cumulative Distribution

The table below gives the probability of that a Poisson random variable X with mean = λ is less than or equal to x. That is, the table gives

$$P(X \le x) = \sum_{r=0}^{x} \lambda^r \frac{e^{-\lambda}}{r!}$$

$\lambda=$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8
$x=$ 0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679	0.3012	0.2466	0.2019	0.1653
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358	0.6626	0.5918	0.5249	0.4628
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197	0.8795	0.8335	0.7834	0.7306
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810	0.9662	0.9463	0.9212	0.8913
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	0.9923	0.9857	0.9763	0.9636
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9985	0.9968	0.9940	0.9896
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9974
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$\lambda=$	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.5	5.0	5.5
$x=$ 0	0.1353	0.1108	0.0907	0.0743	0.0608	0.0498	0.0408	0.0334	0.0273	0.0224	0.0183	0.0111	0.0067	0.0041
1	0.4060	0.3546	0.3084	0.2674	0.2311	0.1991	0.1712	0.1468	0.1257	0.1074	0.0916	0.0611	0.0404	0.0266
2	0.6767	0.6227	0.5697	0.5184	0.4695	0.4232	0.3799	0.3397	0.3027	0.2689	0.2381	0.1736	0.1247	0.0884
3	0.8571	0.8194	0.7787	0.7360	0.6919	0.6472	0.6025	0.5584	0.5152	0.4735	0.4335	0.3423	0.2650	0.2017
4	0.9473	0.9275	0.9041	0.8774	0.8477	0.8153	0.7806	0.7442	0.7064	0.6678	0.6288	0.5321	0.4405	0.3575
5	0.9834	0.9751	0.9643	0.9510	0.9349	0.9161	0.8946	0.8705	0.8441	0.8156	0.7851	0.7029	0.6160	0.5289
6	0.9955	0.9925	0.9884	0.9828	0.9756	0.9665	0.9554	0.9421	0.9267	0.9091	0.8893	0.8311	0.7622	0.6860
7	0.9989	0.9980	0.9967	0.9947	0.9919	0.9881	0.9832	0.9769	0.9692	0.9599	0.9489	0.9134	0.8666	0.8095
8	0.9998	0.9995	0.9991	0.9985	0.9976	0.9962	0.9943	0.9917	0.9883	0.9840	0.9786	0.9597	0.9319	0.8944
9	1.0000	0.9999	0.9998	0.9996	0.9993	0.9989	0.9982	0.9973	0.9960	0.9942	0.9919	0.9829	0.9682	0.9462
10	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9981	0.9972	0.9933	0.9863	0.9747
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994	0.9991	0.9976	0.9945	0.9890
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9992	0.9980	0.9955
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9983
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ= x=	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	11.0	10.0	12.0	14.0	15.0
0	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005	0.0002	0.0005	0.0001	0.0000	0.0000
2	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028	0.0012	0.0028	0.0005	0.0001	0.0000
3	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103	0.0049	0.0103	0.0023	0.0005	0.0002
4	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293	0.0151	0.0293	0.0076	0.0018	0.0009
5	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671	0.0375	0.0671	0.0203	0.0055	0.0028
6	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301	0.0786	0.1301	0.0458	0.0142	0.0076
7	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202	0.1432	0.2202	0.0895	0.0316	0.0180
8	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328	0.2320	0.3328	0.1550	0.0621	0.0374
9	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579	0.3405	0.4579	0.2424	0.1094	0.0699
10	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830	0.4599	0.5830	0.3472	0.1757	0.1185
11	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968	0.5793	0.6968	0.4616	0.2600	0.1848
12	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916	0.6887	0.7916	0.5760	0.3585	0.2676
13	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645	0.7813	0.8645	0.6815	0.4644	0.3632
14	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165	0.8540	0.9165	0.7720	0.5704	0.4657
15	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513	0.9074	0.9513	0.8444	0.6694	0.5681
16	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730	0.9441	0.9730	0.8987	0.7559	0.6641
17	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857	0.9678	0.9857	0.9370	0.8272	0.7489
18	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928	0.9823	0.9928	0.9626	0.8826	0.8195
19	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965	0.9907	0.9965	0.9787	0.9235	0.8752
20	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9953	0.9984	0.9884	0.9521	0.9170
21	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9977	0.9993	0.9939	0.9712	0.9469
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9990	0.9997	0.9970	0.9833	0.9673
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9995	0.9999	0.9985	0.9907	0.9805
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	1.0000	0.9993	0.9950	0.9888
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	0.9997	0.9974	0.9938
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9987	0.9967
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9983
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9991
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000